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MATRIX ANALYSIS IN ECONOMICS AND TOURISM: MODELS, CHALLENGES AND PRACTICAL SOLUTIONS

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Abstract

This paper investigates the application of matrix analysis and linear algebra as fundamental tools for solving complex economic problems, with a specific focus on the tourism sector. Central to this study is the process of mathematical modeling, where real-world economic scenarios—such as supply chain interdependencies (Input-Output models) and market share dynamics (Markov chains) are translated into structured matrix systems. A significant portion of the research is dedicated to the pedagogical and practical challenges faced by students and educators in this field. The transition from abstract algebraic operations, such as matrix inversion and determinants, to their functional interpretation in tourism management (e.g., Revenue Management and cost optimization) represents a major cognitive hurdle. By analyzing these "modeling gaps," the paper suggests that the integration of computational tools and case-study methods is essential for bridging the divide between theory and practice. The findings conclude that mastering matrix-based modeling is not merely a technical requirement but a strategic necessity for future professionals navigating the data-driven landscape of modern tourism.

Key Words: *Matrix Analysis, Linear Algebra, Economic Modeling, Tourism Economics, Input-Output Model, Markov Chains, Optimization*
JEL classification: A22, C02, C61, C67, Z32

Introduction

This section provides a brief literature review of similar problems, modern research, and conclusions regarding the application of linear

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algebra and matrix analysis in non-mathematical studies. In recent decades, increased attention has been given to students' understanding and use of mathematical concepts in various high-level professional studies (e.g., Jones, 2013; Thompson & Silverman, 2008). While matrix calculus is a foundational concept for pure mathematics, it serves as the critical basis for real-world applications in science, engineering, business, and particularly economics and tourism (Barnett et al., 2008). Unfortunately, there remains a significant gap between teaching linear algebra at mathematical faculties versus non-mathematical ones. Applying these concepts in other fields is challenging because variables in matrix expressions and systems of linear equations acquire a new level of meaning (Dray & Manogue, 2005). Furthermore, students often struggle to transfer their mathematical knowledge to their specific field of education, as they do not always see the connection between abstract matrix operations and practical economic reality (Gainsburg, 2006). A primary goal for mathematical educators at non-mathematical faculties should be to cultivate knowledge that facilitates the application, e.g. of matrix concepts in professional domains (Trouche, 2004).

In this paper, we concentrate on teaching the concepts of matrix algebra and systems of linear equations within business, economics, and tourism studies. Previous research has shown that different conceptualizations of mathematical tools can save resources for students and allow for easier application. For instance, understanding a matrix not just as a grid of numbers, but as a transformation of resources or a transition of market states, is highly beneficial for students in tourism management (Chan, 2013.).

Research Focus and Study Area

In this paper, we present several examples of how certain conceptualizations of matrix analysis develop an understanding of problems in economics and tourism. Specifically, we focus on three common interpretations: Linear System and Extended Matrices, Matrix Equation and Applications (Input-Output model) and Matrices as state transition probabilities (Markov chains in tourism loyalty). Calculus and algebra textbooks often focus on formal operations like matrix inversion or determinants, but there is often a lack of practical application in high school curricula (Official Gazette of the Republic of Serbia, 2010). Consequently, many students struggle to move beyond formal, mechanical problem solving. This assertion is supported by teaching

experience, showing that students often "calculate" without understanding the economic implication of the result (Skemp, 1978). The compulsory mathematics courses at faculties of economics in the Republic of Serbia typically consist of 45 hours of lectures and 30 hours of practical exercises. These courses are usually organized as one-semester subjects in the first year of undergraduate studies, with an increasing emphasis on independent student work and additional forms of instruction aimed at ensuring the successful mastery of all topics. However, due to limited teaching time and gaps in prior knowledge acquired in secondary education, students often encounter significant difficulties in understanding abstract mathematical concepts. This is particularly evident in study programs where mathematics is not the primary focus. Therefore, it is necessary to develop teaching approaches that both encourage independent learning and enhance student motivation.

In this paper, we address the following question: how can students enrolled in non-mathematics-oriented study programs develop functional mathematical competencies that will be useful for their future job. To this end, we focus on selected teaching units within the first-semester course Business Mathematics and Statistics at the Faculty of Hotel Management and Tourism in Vrnjačka Banja, University of Kragujevac. Based on our knowledge about the number of enrolled students to the first year of academic studies and the secondary school they attended, a substantial proportion of first-year students at this faculty come from educational backgrounds in which key mathematical topics-particularly those related to more advanced analytical concepts-are either insufficiently covered or entirely absent. As a result, students often perceive these topics as highly abstract and difficult to relate to real-world applications.

To address this issue, this paper proposes the use of application-oriented examples grounded in economic and tourism contexts. We illustrate how matrix analysis and linear algebra can be applied to model real-world problems through linear system and extended matrix, then matrix equation, intersectoral dependencies (Input-Output models), and market dynamics (Markov chains). These examples are designed to be accessible to first-year students while simultaneously bridging the gap between abstract mathematical operations and their practical interpretation. For the research and presentation of results, we used methods of applied linear algebra, descriptive statistics and comments in students' surveys.

Applications for Matrix Analysis in Business and Economics

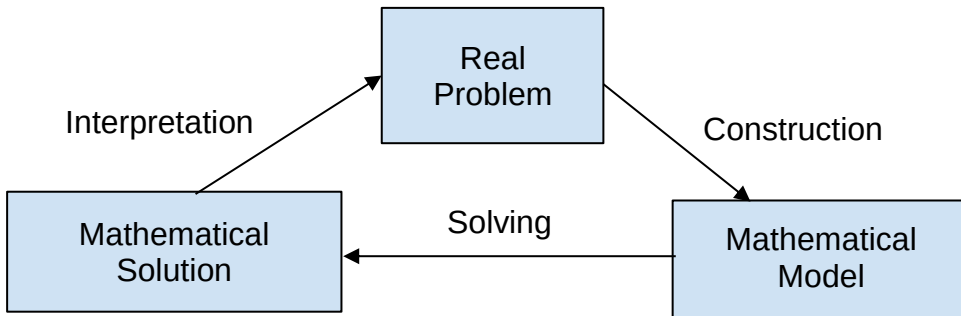
First, we will remind with basic concepts from matrix analysis that students need to be familiar with before encountering their applications in business and tourism models. It is necessary to introduce students with the term of matrix, expanded matrix, to systems of linear equations and matrix operations, which can be interpreted as tools for structuring and solving real-world problems. We can understand the matrix as a square field of arranged numbers separated in columns and rows with the defined positions. For labeling the elements and dimensions of matrices we could use notation $A = [a_{ij}]_{m \times n}$. In general case for system of linear equations given with

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 &= k_1 \\ a_{21}x_1 + a_{22}x_2 &= k_2 \end{aligned}$$

we could join the expanded matrix given with $\left[\begin{array}{cc|c} a_{11} & a_{12} & k_1 \\ a_{21} & a_{22} & k_2 \end{array} \right]$.

In the presented case, we have system with two equations and two variables, and in a similar way, we could construct the expanded matrix for a larger system (with more equations and variables). We also remind students that system could have three possibilities for solutions: unique solution, infinitely many solutions and the system does not have solutions. Then, we will mention three possible forms of the reduced expanded matrix for the system of linear equations with two equations and two variables: $\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 1 & n \end{array} \right]$ (unique solution), $\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 0 & 0 \end{array} \right]$ (infinitely many solution), $\left[\begin{array}{cc|c} 1 & 0 & m \\ 0 & 0 & n \end{array} \right]$ (system does not have a solution) (see Barnett et al., 2006).

In the following example, we will show that systems of equations with more than one solution are suitable for mathematical modelling of the problems from the real world. This process of solving real problems could be separated into three steps (see Figure 1).

Figure 1: *Three steps of mathematical modeling of real problem*

Source: Author's research

Example 1. A logistics company is expanding its fleet and intends to purchase exactly 30 new trucks to achieve a total combined hauling capacity of 33,000 cubic feet. The following three models are available on the market: Compact Mover: 10 feet long with a capacity of 400 cubic feet, Standard Cargo: 14 feet long with a capacity of 800 cubic feet, Maximum Hauler: 26 feet long with a capacity of 1,600 cubic feet. Determine the number of trucks each type of company must buy to satisfy these requirements.

Solution. We will solve this problem using the steps given in Figure 1. Therefore, first we need to determine the variables which specify the number of each type of truck. According to the condition form the task, we have: x_1 is the number of 10-foot trucks; x_2 is the number of 14-foot trucks; x_3 is the number of 26-foot trucks. The corresponding mathematical model is

$$\begin{aligned} x_1 + x_2 + x_3 &= 30 \\ 400x_1 + 800x_2 + 1600x_3 &= 32000 \end{aligned}$$

Solving this model with standard Gauss-Jordan elimination method (see Barnett et al., 2006), we obtain the reduced system

$$x_1 - 2x_3 = -20 \text{ from which we have } x_1 = 2x_3 - 20,$$

$$x_2 + 3x_3 = 50 \text{ from which we have } x_2 = -3x_3 + 50,$$

Let $x_3 = t$, and $t \in R$. The general solution of the system is: $x_1 = 2x_3 - 20$, $x_2 = -3x_3 + 50$ and $x_3 = t$. Since the number of trucks must be a

non-negative integer or zero, we have $t \in N \cup \{0\}$. The first and second equations of the reduced system set additional constraints for t . Now, we have $x_1 = 2t - 20 \geq 0$ implies $t \geq 10$ and $x_2 = -3t + 50 \geq 0$ implies $t \leq 16\frac{2}{3}$. Thus, we obtain the only possible values for t that provide a meaningful solution are from the set $\{10, 11, 12, 13, 14, 15, 16\}$.

One of the most important applications of matrices and inverse matrices is in branch of applied mathematics known as input-output analysis. Input-output analysis deals with the analysis of the connection of the product sector to determine how each sector or subsector (activity, branch) uses the products of other sectors or sub-sectors as elements in the production of their own final product and which product can be used as an element in the products of other subsections. In addition, input-output analysis deals with the analysis of direct and indirect effects of changes in any sector on other sectors.

For the two industries, we have the following input-output model that could be represented in the form of matrix equation

$$X = MX + D.$$

The element m_{ij} of the technological matrix $M = [m_{ij}]_{2 \times 2}$ could be described as the value of the product which industry C_i uses for production of its own product in the value of one unit of currency for the necessity of the industry C_j . The solution of the previous matrix equation is given with

$$X = (I - M)^{-1}D,$$

if the matrix $(I - M)^{-1}$ and $D = [d_{ij}]_{2 \times 1}$ is the matrix of the external requirements.

Remark. For the calculation of the inverse matrix $(I - M)^{-1}$ we could use the method of the expanded matrix of the form $[I - M | I]$ and equivalent transformations on to the rows in order to reduce this matrix to the matrix of the form $[B | I]$, where $B = (I - M)^{-1}$ (see Barnett et al. 2006). Matrix I is the identity matrix of the dimension $n \times n$.

Example 2. Suppose a local tourism economy consists of two primary sectors: Transportation (T) and Accommodation (H). To produce \$1 of Transportation, it requires \$0.10 from its own sector and \$0.20 from Accommodation. To produce \$1 of Accommodation, it requires \$0.30 from Transportation and \$0.10 from its own sector. The economy must satisfy an external (final) demand from international and domestic tourists of \$50 million for Transportation and \$80 million for Accommodation. Find the total production (output) level required for each sector to satisfy both the internal industrial needs and the external final demand.

Solution. From the conditions of the task we have technological matrix $M = \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.1 \end{bmatrix}$ and the matrix of final demand $D = \begin{bmatrix} 50 \\ 80 \end{bmatrix}$. We need to determine X in the model $MX + D = X$. We have $X = (I - M)^{-1}D$, and after a short calculation we obtain

$$(I - M)^{-1} = \frac{1}{0.75} \begin{bmatrix} 0.9 & 0.3 \\ 0.2 & 0.9 \end{bmatrix}$$

and

$$X = \begin{bmatrix} x_T \\ x_H \end{bmatrix} = \begin{bmatrix} 92 \\ 109.33 \end{bmatrix}.$$

The last application we will show is the Markov chains. It represents a powerful combination of probability theory and linear algebra, where the dynamics of a system are described through changes in state at discrete time intervals. The probability of transitioning to the next state depends solely on the current state, not on the history of the system's movement. For transition matrix $P = [p_{ij}]$, each element p_{ij} represents the probability of transitioning from state i to state j . The sum of the elements in each row of the matrix must be exactly 1 (because the system must transition to one of the offered states). State matrix S represents the current probability distribution by states (e.g. market participation). If we keep multiplying the state matrix S by the P matrix, in stable systems we reach a point where participation no longer changes, i.e. $SP = S$.

Example 3. Suppose that we have two destinations: Mountain 1 and Mountain 2. Mountain1: Keeps 80% of its guests, while 20% go to Mountain 2. Mountain 2: Keeps 70% of its guests, while 30% move to

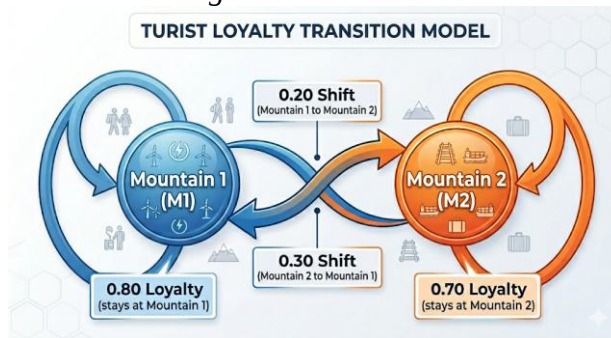
Mountain 1. If Mountain 1 currently has 60% of the market, and Mountain 2 - 40%, what is the situation after one season?

Solution. Following conditions of the task we have state matrix $S = [0.6 \ 0.4]$ and transition matrix $P = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}$. Therefore, we have the new state

$$S_1 = SP = [0.6 \ 0.4] \times \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix} = [0.6 \ 0.4].$$

In this case, the market is in balance. The number of tourists that Mountain 1 loses (20% of its 60% market = 12% of the total market) is exactly offset by the number of tourists coming from Mountain 2 (30% of its 40% market = 12% of the total market). Management can use this data to decide whether to invest more in marketing (to increase loyalty from 0.8 to 0.9) to tip the balance in its favor (see Figure 2.).

Figure 2: State transition diagram



Source: *Author*

Discussion

Matrix analysis and linear algebra play a fundamental role in understanding complex economic systems, offering a structured framework for modeling interdependencies and dynamic processes. Through matrix-based models, such as Input-Output systems and Markov chains, students are able to analyze relationships between variables, simulate real-world scenarios, and interpret equilibrium states. In this way, abstract algebraic concepts gain practical meaning, enabling more accurate analysis and informed decision-making in tourism and business contexts.

To evaluate the effectiveness of the proposed teaching approach at the Faculty of Hotel Management and Tourism in Vrnjačka Banja, University of Kragujevac, we analyzed students' feedback collected through anonymous evaluation surveys for the course Business Mathematics and Statistics. The survey results indicate that students generally have a positive attitude toward the course and its content. In particular, the introduction of matrix-based applications in economic and tourism contexts has contributed to a better understanding of the material and increased student engagement. Student feedback further highlights the importance of connecting abstract mathematical concepts with real-world applications. Several students emphasized that matrix models helped them better understand the purpose of mathematical tools in their field of study, particularly in areas such as tourism planning, cost optimization, and market analysis. At the same time, some students noted that limited time allocated to these topics represents a constraint, especially for those encountering linear algebra concepts for the first time. For future course improvements, it would be beneficial to conduct in-depth qualitative analyses, such as structured student interviews, in order to identify specific learning difficulties and refine teaching methods accordingly. Additionally, aligning course content more closely with students' prior knowledge from secondary education could help reduce the initial learning gap. The integration of modern software tools for visualization and computation should be also introduced as it supports independent learning and enhances conceptual understanding.

Conclusions

In this paper, we have shown that presenting matrix analysis and linear algebra through applications in business and tourism significantly enhances students' understanding of abstract mathematical concepts. By linking theoretical knowledge with real-world problems, students are better able to grasp the purpose and relevance of matrix-based models. The results indicate that students respond positively to this approach, as it allows them to recognize the practical value of mathematical tools in their future professional development. In particular, the use of presented models enables students to understand complex systems, interpret data, and make informed decisions. Furthermore, this approach contributes to bridging the gap between theory and practice, which is one of the main challenges in teaching mathematics to non-mathematical students. The findings suggest that incorporating application-oriented examples and

computational tools into the curriculum is essential for developing functional mathematical competencies.

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